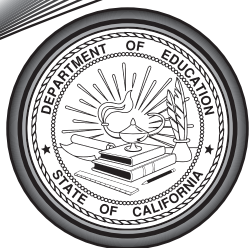


Mathematics

Teacher Guide



**California High School
Exit Examination**



California High School Exit Examination Mathematics Teacher Guide

© 2008 California Department of Education (CDE)

Permission is granted in advance for reproduction of this document for educational purposes. The content must remain unchanged and in its entirety as published by the California Department of Education. To obtain permission to reproduce the information (text or graphics) contained in this document for any commercial purpose, submit the specifics of your request in writing to the Copyright Program Office, California Department of Education, CDE Press, 1430 N Street, Suite 3207, Sacramento, CA 95814. Fax: 916-324-9787

Questions about this document should be directed to CDE's High School Exit Examination Office at 916-445-9449.

Contents

| | |
|--|-----------|
| Introduction | 1 |
| Purpose and Content | 2 |
| Overview of the Strands | 3 |
| 1. Number Sense | 4 |
| 2. Statistics, Data Analysis, and Probability | 15 |
| 3. Algebra and Functions | 27 |
| 4. Measurement and Geometry | 40 |
| 5. Mathematical Reasoning | 57 |
| 6. Algebra I | 66 |
| Appendices | 78 |
| Appendix A Test and Item Development | 79 |
| Appendix B Resources | 82 |
| Appendix C Glossary of Terms Used in This Guide | 83 |

Introduction

The California High School Exit Examination (CAHSEE) Teacher Guide for mathematics is designed to provide comprehensive and accessible information to assist teachers in preparing students for the CAHSEE. Teachers are encouraged to reproduce sections or all of the guide for classroom use. Districts and school personnel are encouraged to use this material in staff development activities.

- “Purpose and Content” provides an overview of the CAHSEE.
- “Overview of the Strands” gives detailed information about how the California academic content standards for mathematics are tested on the CAHSEE. The following mathematics standards are assessed on the CAHSEE:

Number Sense

Statistics, Data Analysis, and Probability

Algebra and Functions

Measurement and Geometry

Mathematical Reasoning

Algebra I

The overview provides a summary of the essential knowledge and skills covered, followed by information about how the standard may be tested. For each standard, a released test question from a previous administration of the CAHSEE is provided with an explanation of the correct answer and an analysis of the incorrect answers.

Purpose and Content

The primary purpose of the CAHSEE is to significantly improve pupil achievement in public high schools and to ensure that pupils who graduate from public high schools can demonstrate grade-level competency in reading, writing, and mathematics. The CAHSEE helps identify students who are not developing skills that are essential for life after high school and encourages districts to give these students the attention and resources necessary to help them achieve these skills during their high school years.

The CAHSEE assesses a range of difficulty levels consistent with good testing practices. Questions assess full mastery of the designated academic content standards as well as foundational knowledge and skills underlying these standards, as recommended by the High School Exit Examination Standards Panel.

In mathematics, standards from grades six and seven and Algebra I are included on the CAHSEE because these academic content standards represent both foundational and competency standards that students should meet to graduate from high school.

All questions on the examination have been evaluated for their appropriateness for measuring the designated mathematics academic content standards. They have been reviewed and approved by committees of California educators, including teachers, administrators, and academicians. All items have also been reviewed and approved by California educators for their adherence to the principles of fairness and have been evaluated to determine if bias exists with respect to characteristics, such as gender, ethnicity, and language.

The test blueprints for the CAHSEE, which indicate the academic content standards tested and the number of items per standard, are available on the CAHSEE Web site.

The CAHSEE is not a timed test, which means it has no fixed time limit in which students must complete the examination. However, students are expected to complete their work during the regular school day unless their Individualized Educational Program (IEP) or Section 504 Plan specifies the need for extra time beyond the school day.

Mathematics Academic Content Standards

As mentioned above, the standards for the mathematics part of the CAHSEE are taken from the California academic content standards in grades six and seven and in Algebra I. They include standards from the following mathematical strands: Number Sense; Statistics, Data Analysis, and Probability; Algebra and Functions; Measurement and Geometry; Mathematical Reasoning; Algebra I.

The mathematics part of the CAHSEE contains 92 multiple-choice test questions (80 operational questions and 12 field-test questions).

Overview of the Strands

The Mathematics part of the CAHSEE assesses designated California academic content standards from grades six and seven and Algebra I. A multiple-choice format is used to assess six strands: Number Sense; Statistics, Data Analysis, and Probability; Algebra and Functions; Measurement and Geometry; Mathematical Reasoning; and Algebra I. Each of these strands is described in detail in the following section of the Teacher Guide. For reporting purposes, the Statistics, Data Analysis, and Probability strands for grades six and seven are combined. The Mathematical Reasoning questions, which are always based on concepts in Number Sense; Statistics, Data Analysis, and Probability; Algebra and Functions; and Measurement and Geometry, are reported under those strands. Thus, there is no reporting category specifically for Mathematical Reasoning.

The CAHSEE focuses on mathematics constructs that are taught and assessed throughout elementary, middle, and high school.

Although questions for the mathematics part of the CAHSEE do not specifically test students on mathematics vocabulary, they may require students to understand mathematical terms. It is especially important that students know the terms that appear in the language of the academic content standards associated with a question.

The following pages of the Teacher Guide discuss the mathematics strands and academic content standards included in the CAHSEE. The mathematics strands are:

- Number Sense
- Statistics, Data Analysis, and Probability
- Algebra and Functions
- Measurement and Geometry
- Mathematical Reasoning
- Algebra I

After each strand is described, each standard in that strand is discussed in detail, and a sample released test question is provided to illustrate each standard. Also included are possible reasons that students might select the distractors, as well as an explanation of the correct answer.

Teachers will find this section of the guide useful in understanding how the California academic content standards are assessed on the CAHSEE. A thorough understanding of the standards and the test questions associated with them will help teachers focus their instruction on the academic content standards and better prepare students for the exam.



Number Sense

14 questions

12 questions

17 questions

17 questions

8 questions

12 questions

**Number
Sense**
**Statistics,
Data
Analysis,
Probability**
**Algebra
and
Functions**
**Measurement
and
Geometry**
**Math
Reasoning**
Algebra I

CAHSEE contains 14 Number Sense items. Students' understanding of fractions, decimals, percents, and integers and their relationship to each other and to the other disciplines of mathematics is an essential component of their mathematics learning. CAHSEE questions in the Number Sense strand require students to demonstrate a foundational understanding of numbers and ways they are represented.

Students will be asked to:

- solve problems with fractions, decimals, and percents.
- compare and order numbers.
- demonstrate an understanding of percents, including those less than 1 and greater than 100.
- understand and meaningfully interpret large and small numbers in scientific notation.
- use specific characteristics of numbers, such as multiples, factors, and primes.
- use and represent integers as the basis for the comparison of quantities.

Essential to success in this CAHSEE strand is the student's understanding of the mathematical operations and the ways they are related to each other. This understanding includes:

- the meaning of arithmetic operations with fractions, decimals, and integers.
- the associative and commutative properties of addition and multiplication.
- the distributive property of multiplication over addition.
- the understanding and use of inverse relationships of addition and subtraction, multiplication, and division.
- finding square roots, squaring numbers, and using the inverse relationship between them.

Students also should possess computational fluency. They should be able to select appropriate methods and tools for computing with fractions and decimals; perform mental arithmetic; use algorithms for computing with fractions, decimals, and integers; use strategies for estimation and for judging the reasonableness of results; and be able to analyze and explain methods for solving problems with proportions.

The ten California academic content standards covered by the CAHSEE Number Sense strand are discussed in the following pages.

| | | |
|----------|--|--|
| Strand | Number Sense (NS) | The radius of the earth's orbit is 150,000,000,000 meters. What is this number in scientific notation? A 1.5×10^{-11} B 1.5×10^{11} C 15×10^{10} D 150×10^9 |
| Standard | 7NS1.1 Read, write, and compare rational numbers in scientific notation (positive and negative powers of 10) with approximate numbers using scientific notation. 1 test question | |

M00213

Scientific notation is required knowledge in science and engineering because many numbers are either so large or so close to zero that there is no other convenient way to write them. CAHSEE questions in this standard require students to demonstrate understanding of the basic concepts of scientific notation using approximations of very large and very small numbers. Test questions may also involve the translation of approximate numbers into scientific notation, the comparison of numbers in scientific notation with either positive or negative exponents, and the understanding of the relative size of two numbers in scientific notation.

Sample Test Question

The correct answer is choice B. Students should recognize that the place-value distance from the 1 (highest place value, 100 billion) to the decimal is 11 digits and that this value provides an appropriate representation of the equivalence as 10^{11} , also equivalent to $10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10$. Students also should know that expressions in scientific notation may include a multiplier, between 1 and 10, along with an exponential value of 10. While exact powers of 10 are expressed without a multiplier (e.g., 10^{11}), numbers such as 150,000,000,000 require a multiplier along with the equivalent power of 10. A typical method of finding the multiplier and the exponent is to count the number of decimal places the decimal must move to create a number between 1 and 10. In the example, the decimal point is moved 11 places to the left to get 1.5 for the multiplier and $+11$ for the exponent.

Analysis of Distractors

Distractor A: expressed the power of 10 as $\frac{1}{10^{11}}$ (students are often unsure of the direction the decimal point should move)

B: correct answer

Distractor C: equivalent in number to 150,000,000,000, but not in scientific notation

Distractor D: equivalent in number to 150,000,000,000, but not in scientific notation

Strand **Number Sense (NS)**

Standard **7NS1.2**

Add, subtract, multiply, and divide rational numbers (integers, fractions, and terminating decimals) and take positive rational numbers to whole-number powers.

3 test questions

John uses $\frac{2}{3}$ of a cup of oats per serving to make oatmeal. How many cups of oats does he need to make 6 servings?

A $2\frac{2}{3}$

B 4

C $5\frac{1}{3}$

D 9

M23015

All students should understand the basic arithmetic functions involving rational numbers in all forms and be comfortable performing calculations with positive and negative numbers. CAHSEE questions in this standard require students to demonstrate computational fluency with rational numbers and an understanding of the relationships between these types of numbers. These skills are fundamental to achievement of the California academic content standards in mathematics.

Sample Test Question

The correct answer is choice B. Students should understand that multiplying the initial amount of oats by the 6 servings yields the easiest calculation.

Analysis of Distractors

Distractor A: added the two numbers and made a mistake in the calculation $\left(\frac{2}{3} + \frac{6}{1} = \frac{8}{3} = 2\frac{2}{3}\right)$

B: correct answer

Distractor C: subtracted $\frac{2}{3}$ from the 6 servings

Distractor D: divided 6 by $\frac{2}{3}$